Passivity-based Angular Rate Feedback Controller for Fin-Controlled Missiles

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Abstract A passivity-based angular rate feedback controller is proposed for a fin-controlled missile system. To augment the stability of the flight systems, rate feedback loop is designed as the inner-loop system. In the flight envelope, aerodynamics dominantly affect the flight dynamics, some of which may be taken advantage of the flight control system. In this study, passive characteristics in the flight dynamics are investigated, and the characteristics are exploited in controller design. The similarities between the classical stability augmented system and the proposed controller are analyzed. Numerical simulation is performed to demonstrate the effectiveness of the proposed controller.

1 Introduction

Passivity-based control (PBC), which stabilizes the system using dissipative and passive characteristics, has been studied for decades. Because the PBC has advantages in perspective of energy-based control([1]), the PBC usually uses Euler-Lagrange dynamics and has been widely used in the field of spacecraft([2, 3, 4]) and robot manipulator([5, 6]). However, few studies have been done for atmospheric flight control systems.

There have been lots of studies for the design of the angular rate controller for the flight control system. Because the translational motion and the rotational motion are strongly coupled in the flight control system, the primary objective of the rate controller is to improve the stability of the rotational motion of the system. Therefore,

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stability augmentation system (SAS) has been widely designed to provide the stability of the system in the flight control system based on the classical control method . Using nonlinear control methodologies, attitude control problem of the missile has been studied utilizing feedback-linearization([7]), backstepping([8]), and sliding-mode control([9]).

To design a PBC, in general, the governing equation was derived based on Euler-Lagrange formulation. Because the flight dynamics is usually described by Newton-Euler formulation, the physical passivity in the dynamics cannot be easily observed. Lee and Kim designed an attitude autopilot using PBC for a fin-controlled missile [10], where the governing equation was derived using Euler-Lagrange formulation.

In this study, a rate-feedback controller for fin-controlled aerodynamic missiles is proposed using the passivity theory. To design the controller, passive characteristics in the missile dynamics is investigated. A rate-feedback controller is designed for the longitudinal motion, and the designed— pitch-rate controller is analyzed by comparing the structure with a classical SAS. Based on the analysis, the rate feedback controller is also designed for roll-pitch-yaw motion. Numerical simulation is performed by comparing the proposed controller with the feedback-linearization control scheme. By the proposed controller using PBC, damping property and the stability of the system can be improved.

This paper is organized as follows. In section 2, the brief review of the passivity control theory is provided. In section 3, the passive characteristics are investigated, and the controller design and the similarity with the classical SAS are discussed. In section 4, numerical simulation is performed to demonstrate the effectiveness of the proposed controller, and conclusion is presented in section 5.

2 Preliminaries

In this section, the preliminary of passivity is briefly reviewed. Let us consider a p-input-p-output system.

$$\dot{x} = f(x) + g(u)$$
$$y = h(x) \tag{1}$$

For the system (1), the following definitions are introduced[[11]].

Definition 1. The system (1) is said to be passive, if there exists a continuously differentiable positive semidefinite function $V(x)$ (called storage function) such that

$$u^T y \geq \dot{V}(x)$$

In addition, it is

- lossless if $u^T y = \dot{V}$
- output strictly passive if $u^T y \geq \dot{V} + y^T \rho(y)$ and $y^T \rho(y) > 0 \; \forall y \neq 0$.
- strictly passive if $u^T y \geq \dot{V} + \psi(x)$ for some positive definite function ψ.

In all cases, the inequality should hold for all (x, u).

Definition 2. The system (1) is said to be zero-state observable, if no solution of $\dot{x} = f(x,0)$ can stay identically in $S = \{s \in R^n | h(x,0) = 0\}$ other than the trivial solution $x(t) = 0$.

Using the definitions of passivity and zero-state observability, the following theorem addresses the stability of the passivity based control.

Theorem 1. *Consider the system* (1). *If the following conditions hold*
   (C1) Eq. (1) *is passive with a radially unbounded positive definite storage function*
   (C2) Zero-state observable,
   then, the origin $x = 0$ can be globally stabilized by $u = -\varphi(y)$, $y^T \varphi(y) > 0$ for all $y \neq 0$.

Theorem 1 is satisfied if there exists radially unbounded storage function V such that $\frac{\partial V}{\partial x} f(x) \leq 0$ for all x, and $y = [\frac{\partial V}{\partial x} g(x)]^T$, which is known as nonlinear version of KYP(Kalman-Yacubovitch-Popov) lemma.

To investigate the passivity of linear system, let us consider a scalar transfer function as follows.

Definition 3. A proper rational transfer function $G(s)$ is positive real, if

- poles of all elements of $G(s)$ are in the left-half plane (LHP).
- for all real w for which jw is not a pole of any element of $G(s)$, the function $2Re[G(jw)]$ is positive semidifinite.
- any pure imaginary pole jw of any element of $G(s)$ is a simple pole and the residue $\lim_{s \to jw}(s - jw)G(s)$ is positive semidefinite Hermitian.

It is strictly positive real (SPR), if $G(s - \varepsilon)$ is positive real.

For example, $G(s) = 1/s$ is PR because it has no poles in $Re(s) > 0$ and has a simple pole at $s = 0$ whose residue is 1. $G(s) = 1/(s+a)$, $a > 0$ is SPR because

$$pole(G(s)) = -a < 0$$
$$Re(G(jw)) = \frac{a}{w^2 + a^2} > 0 \tag{2}$$
$$\lim_{w \to \infty} w^2 Re(G(jw)) = a > 0$$

Using the SPR, the following lemma gives the relation of SPR with the passivity of the system.

Lemma 1. *The linear time-invariant system $\dot{x} = Ax + Bu$, $y = Cx + Du$ with $G(s) = C(sI - A)^{-1}B + D$ is strictly passive if $G(s)$ is SPR.*

On the other hand, if the open-loop system is not passive, a feedback can be used to achieve passivity which is known as feedback passivation.

Definition 4. The system Eq. (1) is called feedback passive, if there exists a following invertible feedback transformation.

$$u = \alpha(\eta, \dot{\eta}) + \beta(\eta, \dot{\eta})v \qquad (3)$$

where α represents a feed-forward function, and v is a virtual control input. Then, the system can be converted as

$$\dot{x} = f(x) + g(x)\alpha(x) + g(x)\beta(x)v$$
$$y = h(x) \qquad (4)$$

Here, α and β are chosen to satisfy

$$\frac{\partial V}{\partial x}(f(x) + g(x)\alpha(x)) \leq 0$$
$$y = \left[\frac{\partial V}{\partial x}g(x)\beta(x)\right]^T \qquad (5)$$

Therefore, the system can be passive from v to y.

Example 1. Consider a dynamic system.

$$\dot{x} = x^2 - 2x + u, \qquad (6)$$

and $y = x$. The system is not strictly passive by showing that $V(x) = x^2$. Using the feedback-passivation, the passivity-based control law can be generated as

$$u_{PBC} = -x^2 + v \qquad (7)$$

From Theorem 1, the feedback system is passive from y to v as follows.

$$\dot{V}(x) = -2x^2 + xv < xv = x\varphi(x) \qquad (8)$$

By choosing $\varphi(x)$ as $-kx$, the system is strictly passive and asymptotically stable. On the other hand, the feedback linearzation control law can also be obtained as

$$u_{FL} = -x^2 + 2x + v \qquad (9)$$

which yields

$$\dot{V}(x) = xv = x\varphi(x) \qquad (10)$$

Difference between u_{PBC} and u_{FL} is that the passivity control input exploits dissipative property in its dynamics. Figure 1 shows the simulation results for the feedback system of example 1. Compared to feedback linearization, the output by the passivity-based controller is stabilized fast by exploiting inherent dynamic characteristics.
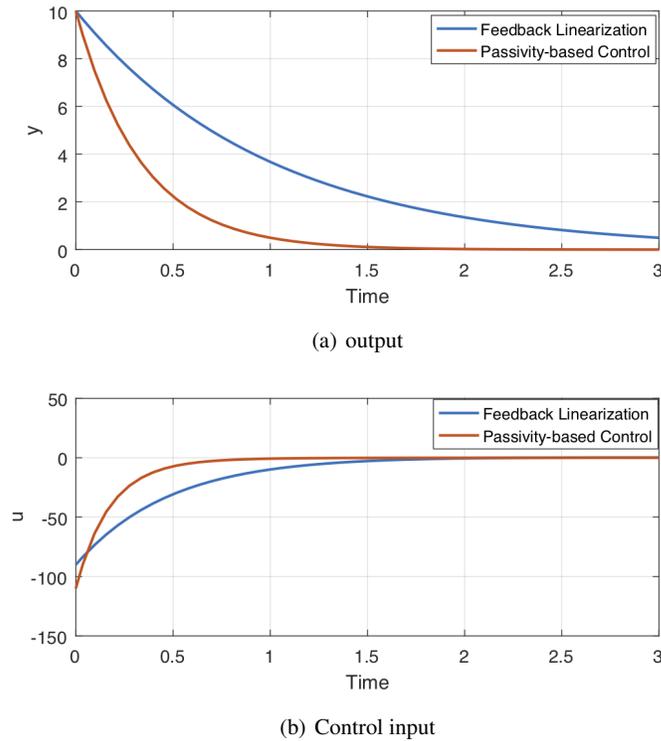
(a) output

(b) Control input

Fig. 1 Simulation results of example 1

3 Controller Design

In this section, the design of rate feedback controller is proposed for a fin-controlled missile. First, the pitch rate controller is designed considering the longitudinal motion of the missile, and the equivalence between the classical SAS is analyzed. Then, the angular rate controller in three-axis is designed in Sec. 3.2.

3.1 Pitch rate controller

Consider a short-period longitudinal motion of the missile which can be described as

$$\dot{\alpha} = \frac{QS}{mV}\left(C_{z_0}(V,\alpha) + C_{z_\delta}\delta\right) + q$$
$$\dot{q} = \frac{QSd}{I_y}\left(C_{m_0}(V,\alpha) + C_{m_q}\frac{d}{2V}q + C_{m_\delta}\delta\right) \qquad (11)$$
$$\dot{\theta} = q$$

where (Q,V) are the dynamic pressure and the speed of the missile, respectively, (S,d) are the reference area and the characteristic length of the missile, respectively, and (m,I_y) indicate the mass and the moment of inertia of the missile. To design a pitch rate controller, the output is introduced as $y = q$. In pitch rate dynamics, the aerodynamic moments have the following properties.

- $M_0 = \frac{QSd}{I_y}C_{m_0}(V,\alpha)$ has a nonlinear function with respect to V and α.
- $M_q = \frac{QSd}{I_y}C_{m_q}$ has positive aerodynamic damping, i.e., $M_q < 0$.
- $M_\delta = \frac{QSd}{I_y}C_{m_\delta}$ is a dimensional moment derivative of control surface, and $M_\delta > 0$.

Using the feedback passivation, the rate-feedback controller based on the passivity-based control is proposed as

$$u_{PBC} = \frac{1}{M_\delta}\left(-M_0 + v\right) \qquad (12)$$

To analyze the stability, let us introduce the storage function $V = \frac{1}{2}y^2$. Then,

$$\dot{V} = q(M_0 + M_q q + M_\delta \delta) = M_q q^2 + qv \qquad (13)$$

Since $M_q < 0$, the system is strictly passive. By selecting $v = \varphi(q) = -Kq$, the controlled system is stabilized as

$$\dot{V} \leq -Ky^2 \qquad (14)$$

3.1.1 Equivalence between the classical stability augmentation system (SAS)

In this section, the connectivity between the passivity-based pitch rate controller and the classical SAS controller is analyzed. First, the classical SAS controller is designed using the linearized system. The dynamic equation (11) can be linearized as

$$\begin{bmatrix} \dot{\Delta\alpha} \\ \dot{\Delta q} \\ \dot{\Delta\theta} \end{bmatrix} = \begin{bmatrix} \bar{Z}_\alpha & 1 & 0 \\ \bar{M}_\alpha & \bar{M}_q & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{M}_\delta \\ 0 \end{bmatrix} \Delta\delta \qquad (15)$$

where $\Delta\alpha = \alpha - \alpha_0$, $\Delta q = q - q_0$, and $\Delta\theta = \theta - \theta_0$ are the perturbed states from the trim condition α_0, $q_0 = 0$, θ_0. Using the linearized system, transfer function $\Delta q/\Delta\delta$ is obtained as

$$\frac{\Delta q(s)}{\Delta\delta(s)} = \frac{\bar{M}_\delta(s - \bar{Z}_\alpha)}{s^2 - (\bar{Z}_\alpha + \bar{M}_q)s + (\bar{Z}_\alpha\bar{M}_q - \bar{M}_\alpha)} \qquad (16)$$

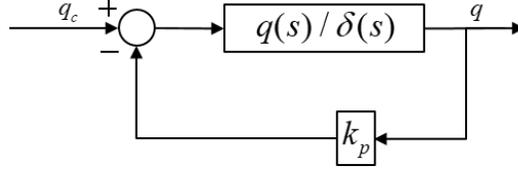As shown in Fig. 2, the classical SAS controller is designed by the pitch rate feed

Fig. 2 Classical rate feedback loop

back.. The closed-loop transfer function can be expressed as

$$\frac{q(s)}{q_c(s)} = \frac{\bar{M}_\delta \left(s - \bar{Z}_\alpha\right)}{s^2 - (\bar{Z}_\alpha + \bar{M}_q - k\bar{M}_\delta)\,s + (\bar{Z}_\alpha\bar{M}_q - \bar{M}_\alpha - k\bar{M}_\delta\bar{Z}_\alpha)} \tag{17}$$

where $\bar{Z}_\alpha < 0$ and $\bar{M}_\alpha < 0$ for statically stable vehicle. The closed-loop pole of the transfer function always lies in the left-half plane (LHP). Now, let us investigate the similarity between the classical control input and the passivity controller. For the trim condition, trimmed control surface δ_0 has

$$\delta_0 = -\frac{1}{\bar{M}_\delta}\bar{M}_0(V_0, \alpha_0) \tag{18}$$

Now, the transfer function $\Delta q/\Delta\delta$ of the system is obtained for the output $y = q$ as

$$\frac{\Delta q(s)}{\Delta \delta(s)} = \frac{\bar{M}_\delta \left(s + z\right)}{s^2 + as + b} \tag{19}$$

where

$$\begin{aligned}
z &= -\bar{Z}_\alpha > 0 \\
a &= -(\bar{Z}_\alpha + \bar{M}_q) > z > 0 \\
b &= (\bar{Z}_\alpha\bar{M}_q - \bar{M}_\alpha) > 0
\end{aligned} \tag{20}$$

Note that the transfer function is SPR, which means that the perturbed system is strictly passive by Lemma 1. Therefore, the system can be asymptotically stabilized by any negative feedback $\Delta\delta = -kp$. The perturbed system is made by the trimmed control input δ_0, and therefore the actual control input u for the classical SAS can be expressed as

$$u = \delta_0 + \Delta\delta = -\frac{1}{\bar{M}_\delta}\bar{M}_0(V_0, \alpha_0) - kp \tag{21}$$

Comparing with the passivity-based pitch rate controller, the classical SAS and the PBC have common structure as

$$u = A - Bq \tag{22}$$

In the classical SAS, A and B are expressed as

$$A = -\frac{1}{\bar{M}_\delta}\bar{M}_0(V_0, \alpha_0), B = k \tag{23}$$

Likewise, in the PBC controller, A and B are

$$A = -\frac{M_0}{M_\delta}, B = K/M_\delta \tag{24}$$

Note that $\bar{M}_\delta$, $\bar{M}_0$ are trimmed coefficients of M_δ and M_0, respectively. Considering $K = M_\delta k$, it can be stated that the passivity-based rate controller is the extended version of the classical SAS.

3.2 Rate-feedback Controller for roll-pitch-yaw axis

In this section, a rate-feedback controller is designed for roll-pitch-yaw integrated system using the passive characteristics in rotational motion. The rotational motion is described by Newton-Euler formulation as ([9])

$$\mathbf{J}\dot{\omega} + \omega \times (\mathbf{J}\omega) = \mathbf{M_0} + \mathbf{M}_\omega \omega + \mathbf{M_u}u \tag{25}$$

where $\omega = [p\,q\,r]^T$ denotes the the angular rate vector, u is the control vector, and $\mathbf{J}$ is the moment of inertia matrix, which can be expressed as

$$\mathbf{J} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \tag{26}$$

where (I_{xx}, I_{yy}, I_{zz}) represent the moments of inertia in the principal axis (x,y,z axis). The aerodynamic moments $(\mathbf{M_0}, \mathbf{M}_\omega, \mathbf{M_u})$ are modeled as

$$\mathbf{M_0} = QSd\begin{bmatrix} C_{l_0} \\ C_{m_0} \\ C_{n_0} \end{bmatrix}, \qquad \mathbf{M}_q = \frac{QSd^2}{2V}\begin{bmatrix} C_{l_p} & 0 & 0 \\ 0 & C_{m_q} & 0 \\ 0 & 0 & C_{n_r} \end{bmatrix}$$

$$\mathbf{M}_u = QSd\begin{bmatrix} C_{l_{\delta_r}} & C_{l_{\delta_p}} & C_{l_{\delta_y}} \\ 0 & C_{m_{\delta_p}} & 0 \\ 0 & 0 & C_{n_{\delta_y}} \end{bmatrix} \tag{27}$$

where $u = [\delta_r\,\delta_p\,\delta_y]^T$ is the fin-deflection vector defined as control input, $(C_{l_0}, C_{m_0}, C_{n_0})$ are the biased aerodynamic moment coefficients, $(C_{l_p}, C_{m_q}, C_{n_r})$ are the aerodynamic damping coefficients, and $(C_{l_{\delta_r}}, C_{l_{\delta_p}}, C_{l_{\delta_y}}, C_{m_{\delta_p}}, C_{n_{\delta_r}})$ are the control derivatives of fin-deflections.

In Eq. (25), the following condition satisfies

$$\omega^T \left(\omega \times (\mathbf{J}\omega)\right) = 0 \tag{28}$$

$$\mathbf{M_\omega} < 0 \tag{29}$$

Using the dynamic characteristics, the dynamic equation can be converted by introducing the feed-forward function α and β as

$$\begin{aligned}
\alpha(\eta, \dot{\eta}) &= \mathbf{M_u}^{-1}(-\mathbf{M_0}) \\
\beta(\eta, \dot{\eta}) &= \mathbf{M_u}^{-1}
\end{aligned} \tag{30}$$

By substituting Eq.(3) and Eq. (30) into Eq. (25), we have

$$\mathbf{J}\dot{\omega} + \omega \times (\mathbf{J}\omega) = \mathbf{M_\omega}\omega + v \tag{31}$$

To show the passivity of the system, let us consider the following storage function.

$$V_{storage} = \frac{1}{2}\omega^T \mathbf{J}\omega \tag{32}$$

Differentiating the storage function with respect to time gives

$$\begin{aligned}
\dot{V}_{storage} &= \omega^T (\mathbf{J}\dot{\omega}) \\
&= \omega^T (-\omega \times (\mathbf{J}\omega) + \mathbf{M_\omega}\omega + v) \\
&\leq y^T v
\end{aligned} \tag{33}$$

which yields

$$V_{storage} - V_{storage}(0) \leq \int_0^t y(s)^T u(s) ds \tag{34}$$

Therefore, the system is strictly passive. Considering $\varphi(y) = -ky = -k\omega$, the system can be stabilized.

Now, let us design a rate-feedback controller based on passivity-based control. To track a desired angular rate ω_d, the error dynamics $e = \omega - \omega_d$ can be rewritten as

$$\mathbf{J}\dot{e} + e \times (\mathbf{J}\omega) = -\mathbf{J}\dot{\omega}_d - \omega_d \times (\mathbf{J}\omega) + \mathbf{M_0} + \mathbf{M_\omega}\omega + \mathbf{M_u}u \tag{35}$$

Feedback control law is considered to stabilize the error dynamics as

$$u = \alpha(\dot{\omega}_d, \omega_d, \omega, \dot{\omega}) + \beta(\omega, \dot{\omega})v \tag{36}$$

In Eq. (36), α, β, and v can be designed as follows,

$$\begin{aligned}
\alpha(\dot{\omega}_d, \omega_d, \omega, \dot{\omega}) &= -\mathbf{M_u}^{-1}\mathbf{M_0} \\
\beta(\omega, \dot{\omega}) &= \mathbf{M_u}^{-1} \\
v &= \mathbf{J}\dot{\omega}_d + \omega_d \times (\mathbf{J}\omega) - \mathbf{M_\omega}\omega_d - K_p\dot{e}
\end{aligned} \tag{37}$$

where K_p is a feedback gain. Using the feedback passivation, the error dynamics can be rearranged as

$$\mathbf{J}\dot{e} + e \times (\mathbf{J}\omega) = \mathbf{M}_\omega e - K_p e \tag{38}$$

To analyze the stability of the passivity-based control system, let us consider a following storage function.

$$V_{storage} = \frac{1}{2} e^T \mathbf{J} e \tag{39}$$

Time derivative of (39) gives

$$\dot{V}_{storage} = e^T \mathbf{J}\dot{e} = e^T \left(-e \times (\mathbf{J}\omega) + \mathbf{M}_\omega e - K_p e\right) \tag{40}$$

Since $a^T (a \times b) = 0$ and $\mathbf{M}_\omega < 0$, Eq. (40) can be rewritten as

$$\dot{V}_{storage} = e^T \mathbf{M}_\omega e - e^T K_p e < -e^T k_p e \tag{41}$$

Therefore, the closed-loop system is strictly passive. By taking $y = e$ and $v_2 = -K_d e$, Eq. (41) is stabilized by Theorem 1.

4 Numerical Simulation

To demonstrate the performance of the proposed angular rate controller, numerical simulation is conducted. Initial conditions of the missile are summarized in Table 1. To compare the tracking performance of the proposed controller, feedback-linearization controller is designed for the same simulation scenario. The control law can be expressed as

$$u_{FL} = \mathbf{M_u}^{-1} \left(-\mathbf{M_0} + e \times (\mathbf{J}\omega) + \mathbf{J}\dot{\omega}_d - \mathbf{M}_\omega \omega - K_p e\right) \tag{42}$$

Note that the gain K_p is selected as 5 in both the proposed controller and feedback linearization controller.

Table 1 Initial Condition

Parameter	Symbol	Units
Initial Position	(x_0, y_0, z_0)=(0,0,10)	(m)
Initial Attitudes	$(\phi_0, \theta_0, \psi_0)$=(0,45,0)	(deg)
Initial Angular rates	(p_0, q_0, r_0)=(0,5,0)	(deg/s)
	Boost-phase	Glide-phase
Speed(Initial, range) $V \in [V_0, V_f]$	$V(t_0) = V_f$	(m/s)
Inertia matrices $\mathbf{J} \in (\mathbf{J}_0, \mathbf{J}_f)$	$\mathbf{J} = \mathbf{J}_f$	

Figures 3 and 4 show the simulation results. In the simulation, both controllers track the desired command. However, the proposed passivity-based controller shows small tracking errors in pitch and yaw axes. Also, the passivity-based controller
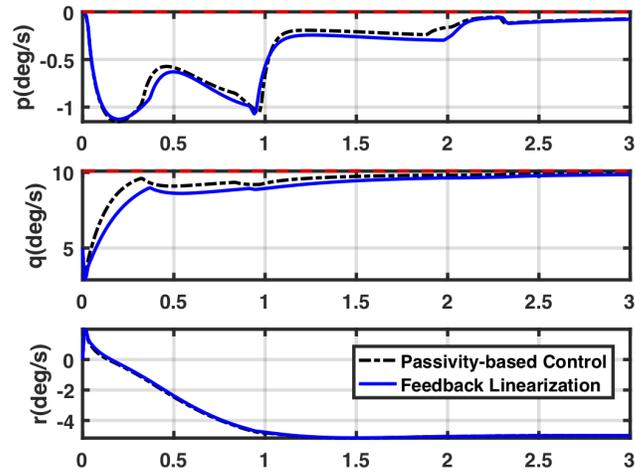
Fig. 3 Time histories of angular rates (p, q, r)
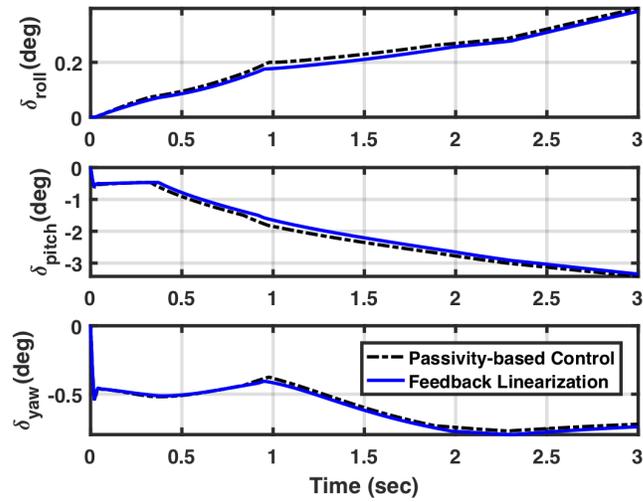

Fig. 4 Time histories of fin deflections $(\delta_r, \delta_p, \delta_y)$

converge to the command faster than the feedback linearization controller, which improves damping property.

To show the robustness of the proposed controller, Monte-Carlo simulation is performed under various uncertainties. For the uncertainty, the aerodynamic coefficient considering bias error is modeled as

$$C_{(\cdot)} = C_{(\cdot),n} \left(1 + \mu_d\right) \tag{43}$$

where $C_{(\cdot),n}$ is the nominal aerodynamic coefficient obtained from the aerodynamic data, μ_d is the bias uncertainty. In the siumlation, 100 cases of aerodynamic uncertainties, $\mu_{d,i} \in [-0.2, 0.2]$, $i = 1, \cdots, 100$, are considered. To compare the performance, two performance indices are considered as

$$J_1 = \frac{1}{T} \int_0^T |p - p_c| + |q - q_c| + |r - r_c| dt \tag{44}$$

$$J_2 = \frac{1}{T} \int_0^T |\delta_r| + |\delta_p| + |\delta_y| dt \tag{45}$$

where J_1 and J_2 represent an average cumulative error and fuel consumption, respectively.

Figure 5 shows the Monte-Carlo simulation result. Under the various uncertainties, both controllers show response deviations from the nominal cases. However, the proposed controller has relatively small deviation and shows consistent tracking performance compared to the feedback linearization controller. Moreover, the worst case of the pitch rate response shows better behavior than the result of the nominal feedback linearization controller as shown in Fig. 5. Table 2 summarizes the result of the performance indices. It is shown that the maximum cumulative error of the proposed method does not exceed the mean value of the feedback linearization, and the standard deviation is much smaller. From the result, the passivity-based rate feedback controller improves the stability of the system.

5 Conclusion

In this study, a passivity-based rate feedback controller was proposed for an atmospheric flight control system. In the rotational dynamics, the dissipative moment could improve its stability, which was demonstrated based on the passivity control theory. The proposed angular rate controller was designed by utilizing the dissipative moment. It was also shown that the structure of the passivity based rate feedback controller was similar to that of the classical stability augmentation system. Simulation result demonstrated that the proposed passivity-based controller improved the stability and showed robust performance.
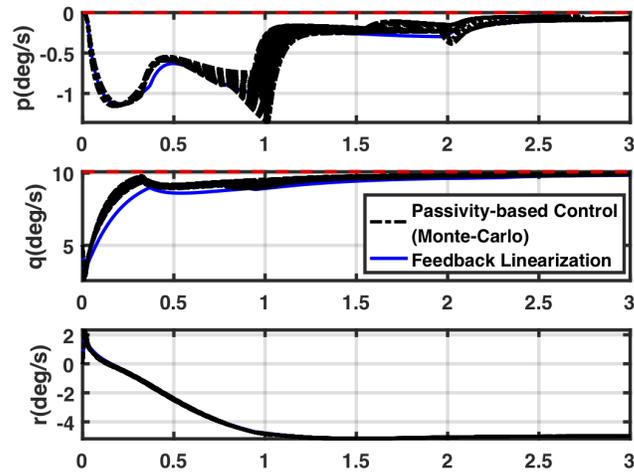
Acknowledgments

Fig. 5 Time histories of angular rates (p, q, r) (Monte-Carlo Simulation, Passivity Based Controller)

Table 2 Simulation results : Monte-Carlo Simulation

	Cumulative error			Fuel Consumption		
	mean	std	max	mean	std	max
Passivity-based Controller	0.0117	0.0014	0.0144	0.0514	0.0034	0.0574
Feedback Linearization	0.0179	0.0021	0.0217	0.0495	0.0030	0.0548

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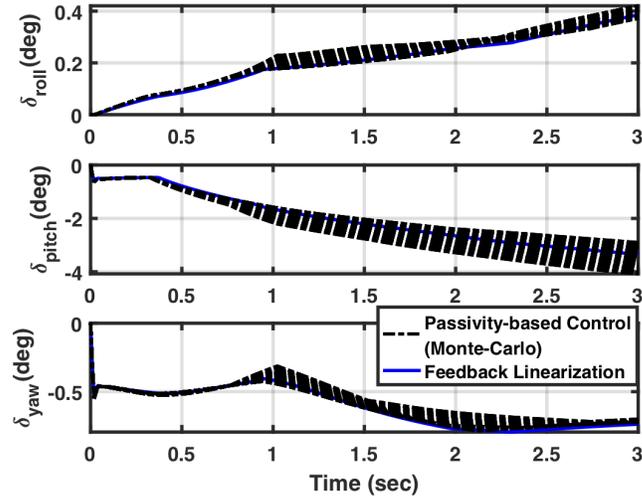
Fig. 6 Time histories of fin deflections $(\delta_r, \delta_p, \delta_y)$ (Monte-Carlo Simulation, Passivity Based Controller)

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